**BUAD 5022 – Individual Problem Set 2**

Due: November 6 by 11:59pm via Blackboard submission

Grade: 15% of your overall class grade

Category A Assignment (i.e., do not discuss with anyone else except the instructor)

*Homework should be neat and organized. It is subject to grade penalties if it is not.*

1. Create a Linear Programming model in Python that minimizes the sum of absolute errors for the following constraints:

4x1 + x2 – x3 + 2x4 + 3x5 + 6x6 = 100 (1)

3x1 + 2x2 + x3 + 4x5 + x6 = 80 (2)

x1 + x2 + x3 + x4 + x5 + x6 = 60 (3)

2x1 + 3x2 + 5x3 – 2x4 + 4x6 = 120 (4)

x2 + x3 + 3x4 + x5 + 2x6 = 40 (5)

3x1 + 5x2 + 2x3 + x4 – 2x5 + x6 = 90 (6)

3x1 – 5x2 – 7x3 + x4 + 9x5 + x6 = 30 (7)

* Formulate the model as an LP.

Introduce variables x7-x13

Minimize

Subject To:

(1)

(2)

(3)

(4)

(5)

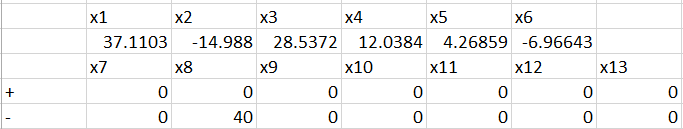
(6)

(7)

(Non-negativity)

* Solve the model using Python (submit your code).
* Summarize the solution in a succinct and presentable manner.

***Note, x1 through x6 are free variables (i.e., they can be positive, zero, or negative).***



The optimal objective is 40, therefore the sum of absolute error is minimized at 40.

2. Joe’s Lumber Company manufacturers three grades of plywood. The following table summarizes the production requirements per unit in each of the three production operations (departments) along with other pertinent information. In addition to the tabulated data, Joe’s Lumber Company has a **combined weekly goal of producing at least 400 units of plywood**. Formulate a linear program to determine how many units of each grade of plywood to manufacture to maximize profit. Assume it is OK to manufacture fractional units (i.e., do not use integer variables). *Hint: You only need 3 variables.*

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Plywood | Profit Per Unit | Processing Time in Hours per Unit | | | Minimum | Maximum |
| Grade | ($/Unit) | Cutting | Lamination | Finishing | Demand | Demand |
| Grade A | 30 | 1.1 | 1.4 | 1.7 | None | 150 |
| Grade B | 20 | 1.2 | 1.5 | 1.8 | None | 200 |
| Grade C | 10 | 1.3 | 1.6 | 1.9 | 100 | None |
| Capacity (Hours Per Week) | | 700 | 600 | 800 | - | - |

*Note, if you use variable restrictions in Python when you define your variables for Minimum and Maximum values; then the reduced costs will be associated with those restrictions. If you choose to use constraints in Python for Minimum and Maximum values, then the shadow prices will be associated with those restrictions.*

Let x1, x2, x3 be number of grade A, B, C plywood produced

Maximize 30x1+20x2+10x3

Subject To:

(Cutting time limit)

(Lamination time limit)

(Finishing time limit)

(Max demand of Grade A)

(Max demand of Grade B)

(Min demand of Grade C)

(combined weekly goal)

(Non-negativity)

status = Optimal

Objective=8566.6666

x1 = 150.0 Reduced Cost = 11.333333

x2 = 153.33333 Reduced Cost = -0.0

x3 = 100.0 Reduced Cost = -11.333333

name shadow price slack

0 C1 -0.000000 221.00000

1 C2 13.333333 -0.00000

2 C3 -0.000000 79.00000

3 C4 -0.000000 -3.33333

* Answer the following questions using your output.

Questions:

At the optimal solution:

1. Of the three departments (Cutting, Lamination, Finishing), which of them are operating at full capacity? (Explain how you know this answer based on your output from Python.)

Lamination department is operating at full capacity since slack for lamination constraint is 0.

1. Of the three departments (Cutting, Lamination, Finishing), for those that are operating at full capacity – how much would you be willing to pay for an additional hour per week of that resource? (Explain how you know this answer based on your output from Python.)

The shadow price of C2 indicates that I am willing to pay 13.333 dollars for an additional hour per week of lamination.

1. For the minimum and maximum demand restrictions, what is the interpretation of either the shadow price or reduced cost for each of those three restrictions? (Explain how you know this answer based on your output from Python.)

For grade A plywood, the reduced cost of 11.33 means that by producing one more product of grade A, the profit would increase by 11.33 dollars.

For grade C plywood, the reduced costs of -11.33 indicates that by producing one more product of grade C, the profit would decrease by 11.33 dollars.

3. Write the general dual problem associated with the given LP.

*(Do not transform or rewrite the primal problem before writing the general dual)*

Minimize 4x1 + 2x2 – x3

Subject To

x1 + x2 + x3 = 20

2x1 – x2 ≥ 6

3x1 + 2x2 + x3 ≤ 4

x1, x2 ≥ 0, x3 unrestricted

Dual:

Maximize

Subject To:

4. Take question 3 from Problem Set 1 *(it is provided below, along with its formulation for your convenience)*. Solve the problem using Python. Fully explain (interpret) the Reduced Costs of each variable, and fully explain the Shadow Prices and Slack Amounts for each constraint.

* Formulate the model as an LP.
* Solve the model using Python (submit your code).
* Summarize the solution in a succinct and presentable manner.
* Fully explain (interpret) the Reduced Costs of each variable, and fully explain the Shadow Prices and Slack Amounts for each constraint.

*Repeated from Problem Set 1:*

Three different investment options are available at the beginning of each year during the next 6-year period. The durations of the investments are 1 year, 3 years, and 5 years. The 1-year investment yields a total return of 5.1%, the 3-year investment yields a total return of 16.2%, and the 5-year investment yields a total return of 28.5%. If an initial investment of $10,000 is made and all available funds are invested at the beginning of each year, formulate a linear programming model to determine the investment pattern that results in the maximum available cash at the end of the sixth year.

* Formulate the model as an LP.

(You do not need to solve. Just write or type the formulation.)

***Formulation:***

Indexed Set:

i = beginning of year i (i = 1, …, 6)

Variables:

wi = amount invested in the one-year investment at the beginning of year i

xi = amount invested in the three-year investment at the beginning of year i

yi = amount invested in the five-year investment at the beginning of year i

*Assume we do not have to create variables that would take us beyond end of year 6.*

Maximize (End of Year 6 Cash On Hand)

1.051w6 + 1.162x4 + 1.285y2

Constraints:

Equilibrium per time period:

w1 + x1 + y1 = 10,000

w2 + x2 + y2 = 1.051w1

w3 + x3 = 1.051w2

w4 + x4 = 1.051w3 + 1.162x1

w5 = 1.051w4 + 1.162x2

w6 = 1.051w5 + 1.162x3 + 1.285y1

Non-negativity:

wi, xi, yi >= 0 for all i

status = Optimal

Objective=13505.349999999999

w1 = 10000.0 Reduced Cost = 0.0

w2 = -0.0 Reduced Cost = 0.0

w3 = 0.0 Reduced Cost = -0.0013830999

w4 = 0.0 Reduced Cost = -0.001064349

w5 = 0.0 Reduced Cost = 0.0

w6 = 0.0 Reduced Cost = 0.0

x1 = 0.0 Reduced Cost = -0.000291

x2 = 0.0 Reduced Cost = -0.001453638

x3 = 0.0 Reduced Cost = -0.0013830999

x4 = 0.0 Reduced Cost = 0.0

y1 = 0.0 Reduced Cost = 0.0

y2 = 10510.0 Reduced Cost = 0.0

name shadow price slack

0 \_C1 1.350535 -0.0

1 \_C2 1.285000 -0.0

2 \_C3 1.222645 -0.0

3 \_C4 1.162000 -0.0

4 \_C5 1.104601 -0.0

5 \_C6 1.051000 -0.0

**Reduced costs**

Both 0 in variable and reduced costs for variables w2, w5, w6, x4 and y1 indicate there are alternate optimal solutions. W1 and y2 have reduced cost of 0 means that they are at their best solutions. W3 has a reduced cost of -0.00138, means that if invest 1 dollar into one-year investment at the beginning of year 3, the maximized profit would decrease by 0.00138 dollars.

For w4, if invest 1 dollar into one-year investment at the beginning of year 4, the maximized profit would decrease by 0.00106 dollars.

For x1, if invest 1 dollar into three-year investment at the beginning of year 1, the maximized profit would decrease by 0.00029 dollars.

For x2, if invest 1 dollar into three-year investment at the beginning of year 2, the maximized profit would decrease by 0.00138 dollars.

For x3, if invest 1 dollar into three-year investment at the beginning of year 3, the maximized profit would decrease by 0.00138 dollars.

**Shadow prices**

The shadow price in year 1 is 1.35, indicates that I am willing to pay $1.35 for 1 dollar increase in my profit in year 1.

In year 2, the profit would be increased by $1.285 if I have 1 more dollar to invest into the portfolio.

In year 3, the profit would be increased by $1.2226 if I have 1 more dollar to invest.

In year 4, the profit would be increased by $1.162 if I have 1 more dollar to invest.

In year 5, the profit would be increased by $1.1046 if I have 1 more dollar to invest.

In year 6, the profit would be increased by $1.051 if I have 1 more dollar to invest.

**Slack**

All slacks are 0 means that all funds are used up in each year.